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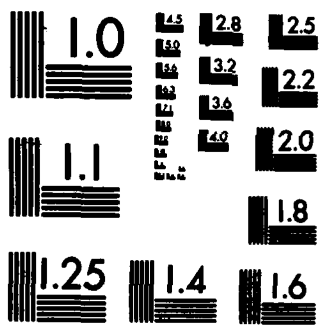
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DUAL ADAPTIVE CONTROL BASED UPON SENSITIVITY FUNCTIONS*

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ABSTRACT

A new adaptive dual control solution is presented for the control of a class of multi-variable input-output systems. Both rapidly varying random parameters and constant but unknown parameters are included. The new controller modifies the cautious control design by numerator and denominator correction terms. This controller is shown to depend upon sensitivity functions of the expected future cost. A scalar example is presented to provide insight into the properties of the new dual controller. Monte-Carlo simulations are performed which show improvement over the cautious controller and the Linear Feedback Dual Controller of [1] and [2].

1. INTRODUCTION

Multi-variable systems which are characterized by uncertain parameters with large random variations are a difficult challenge for most control design techniques. The assumed randomness of the parameter variations often precludes the use of gain scheduling (non adaptive) control design. Stochastic adaptive control theory provides a principal design approach for systems of this type. Exact solution of the stochastic problem with unknown parameters requires solution of the Stochastic Dynamic Programming equation and this is not feasible for practical implementation. The solution is known to have a dual effect [1,2] that can be used to enhance the real-time identification of system parameters as well as provide good control.

Many suboptimal dual solutions have been suggested [1,2,5-11]. The various approaches which have incorporated this dual property can be loosely divided into two classes. In the first class [5-8], the optimal control problem is reformulated to consist of a one-step ahead criterion to be minimized, augmented by a second term which penalizes the cost for poor identification. This approach is attractive due to the analytical tractability of the solution; however, the solution is based on a one-step criterion and does not fully exploit the dual property of a multi-step solution. Padilla and Cruz [14] give a dual control solution for such a plant by minimizing the control objective function subject to an upper bound in the total estimation cost. Their objective function includes a standard control objective function and also a second constraint term which reflects the sensitivity of the parameters to the state of the system. Thus the solution adjusts itself to exercise better estimation for such sensitive parameters within the upper bound. The second class [9-11] utilizes the stochastic dynamic programming equation directly and performs linearization of the future cost in order to obtain a solution. Previous control solutions among this second class require a numerical search procedure which poses difficulties for a practical solution for on-line control for multivariable systems.

The linear feedback dual controller of [1,2] is based upon a first order Taylor series expansion of the expected future cost and is called the first order dual (FOD). It offers some improvement over the non dual cautious control based upon a one-step criterion. The results are based upon a simulation model with constant but unknown parameters. Although the dual control offers some improvement over the cautious controller the improvement is not significant for most practical applications where the system contains constant parameters and the objective is to control in steady state operation. However, for random parameter variations, dual control can sometimes offer significant improvement over non-dual controllers [5,9]. The FOD of [1,2] is attractive due to its simplicity (it is comparable to the cautious control design in algorithm complexity and does not require numerical search). The objective of the present study is to evaluate the cautious controller and the FOD for large random parameter variations modeled as a random walk. Monte-Carlo simulations are performed and conditions quantified under which the dual controller offers significant improvement over a non-dual cautious controller.

The FOD, although offering a reduction in the average cost, is found to be unacceptable in many cases. This is attributed to the sensitivity of the expected future cost whenever the system is characterized by limited controllability. A second order expansion of the linearization procedure of [1,2] is presented to account for this sensitivity. This new second order dual controller (SOD) inherently includes a robustness property in that the controller accounts for sensitivity of the expected future cost due to parameter estimates and their uncertainty. Simulations are presented which show the improvement of the SOD over the cautious controller and the FOD. This SOD uses a Newton type search procedure and is developed for multi-variable systems. One of the main advantages of the SOD presented herein is that it modifies the cautious controller with a numerator "probing" term and a denominator correction term. Although the SOD is still considered too complex for practical implementation, the structure of the control solution is in a form which permits practical design changes to the cautious controller to include the dual properties.

Section 2 gives the problem formulation. The approximate dual controller for the multi-variable input-output system is developed in Section 3. Section 4 analyses this dual controller for a scalar example with one unknown parameter. Section 5 concludes the paper.

2. PROBLEM FORMULATION

The multivariable system under investigation is

$$x(k+1) = c(k) + B(k)\, u(k) \qquad (2.1)$$

where $c(k)$ is an unknown vector and $B(k)$ is a matrix of unknown parameters. The unknown elements of $c(k)$ and $B(k)$ are denoted as $\theta(k)$ with covariance matrix $P(k)$. These are represented by a discrete random model

$$\theta(k+1) = A\theta(k) + v(k) \qquad (2.2)$$

$$E(v(k))=0 \quad \text{and} \quad E(v(k)v'(j)) = V\,\delta_{kj} \qquad (2.3)$$

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The measurement equation is

$$y(k) = x(k) + w(k) \tag{2.4}$$

where

$$E(w(k)) = 0 \quad \text{and} \quad E(w(k)w'(j)) = W\,\delta_{kj}$$
$$E(w(k)v'(j)) = 0 \tag{2.5}$$

and $x(k)$, $y(k)$ being n dimensional vectors. The control criterion to be minimized is the expected value of the cost from step 0 to N

$$J(0) = E\{C(0)\} = E\left\{\sum_{k=1}^{N} x'(k)Qx(k) + u'(k-1)Ru(k-1)\right\} \tag{2.6}$$

where $N = 2$ for the two step ahead criterion.

3. APPROXIMATE DUAL CONTROLLER FOR TWO STEP CRITERION

The minimization of (2.6) with respect to $u(0)$ and $u(1)$ subject to (2.1) - (2.5) is obtained from the Stochastic Dynamic Programming equation [12,13]

$$J^*(k) = \min_{u(k)} E\{C(k)+J^*(k+1)\,|Y^k\} \quad k=N-1,\ldots,1,0 \tag{3.1}$$

where $J^*(k)$ is the "cost-to-go" from k to N and Y^k is the cumulated information at time k when the control $u(k)$ is to be determined. For $N = 1$, (3.1) is

$$J^*(0) = \min_{u(0)} E\{x'(1)Qx(1)+u'(0)Ru(0)+J^*(1)\,|Y^o\} \tag{3.2}$$

where $J^*(1)$ is the optimal cost at the last step and is obtained by minimization of $J(N-1)$ for $N = 2$. Assuming diagonal $Q = \mathrm{diag}(q_\ell)$ this results in [1,2]

$$\begin{aligned} J^*(1) = {} & \hat{c}'(1)Q\hat{c}(1) + \sum_{\ell=1}^{n} q_\ell P_c^\ell(1) \\ & - [\hat{c}'(1)Q\hat{B}(1) + \sum_{\ell=1}^{n} q_\ell P_{cB}^\ell(1)]\,[\hat{B}'(1)Q\hat{B}(1) + \\ & + \sum_{\ell=1}^{n} q_\ell P_B^\ell(1) + R]^{-1}\cdot [\hat{B}'(1)Q\hat{c}(1) + \sum_{\ell=1}^{n} q_\ell P_{Bc}^\ell(1)] \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} u^*(1) = {} & -[\hat{B}'(1)Q\hat{B}(1) + \sum_{\ell=1}^{n} q_\ell P_B^\ell(1) + R]^{-1}[\hat{B}'(1)Q\hat{c}(1) \\ & + \sum_{\ell=1}^{n} q_\ell P_{Bc}^\ell(1)] \end{aligned} \tag{3.4}$$

where

$$P^\ell(1) = \begin{bmatrix} P_c^\ell(1) & P_{cB}^\ell(1) \\ P_{Bc}^\ell(1) & P_B^\ell(1) \end{bmatrix} \tag{3.5}$$

$P(1)$ is the expected value of $(\theta(1))^2$ for time step 2 given measurement $y(1)$ at time step 1. The index ℓ is used to represent the row number in (2.1) and $P^\ell(1)$ is the associated parameter covariance.

The parameter estimates $\hat{\theta}(1)$ and covariances $P(1)$ are obtained from the Kalman filter. Since W is diagonal one can decouple the estimation. Then

$$\hat{\theta}^\ell(1) = A\hat{\theta}^\ell(0)+AK_\ell(1)\,\nu_\ell(1) \tag{3.6}$$

$$K_\ell(1) = P^\ell(0)H'(1)\,[H(1)P^\ell(0)H'(1)+W_\ell]^{-1} \tag{3.7}$$

$$\tilde{P}^\ell(1) = P^\ell(0) - K_\ell(1)H(1)P^\ell(0) \tag{3.8}$$

$$P^\ell(1) = A\tilde{P}^\ell(1)A' + V \tag{3.9}$$

where

$$\nu_\ell(1) = y_\ell(1) - H(1)\hat{\theta}^\ell(0) \tag{3.10}$$

$$H(1) = [1 \;\; u^T(0)] \tag{3.11}$$

$$\theta^\ell(1) = [c_\ell(1) \;\; B_\ell(1)]^T, \quad \ell=1,2,\ldots n \text{ row of B} \tag{3.12}$$

As discussed in [1] and [2] $J^*(1)$ is a nonlinear function of the parameter estimates $\hat{\theta}(1)$ and covariances $P(1)$ and thus a linearization was performed. In [1] a scalar formulation was presented and a first order linearization was performed about the nominal parameter estimate squared $(\hat{\theta}(0))^2$ and nominal covariance $\bar{P}(1)$. Also in [1,2] the vector case was presented and linearization to first order performed. To more accurately account for the dual effect a second order Taylor Series expansion is presented about $\hat{\theta}(0)$ and a first order expansion about the nominal covariance $\bar{P}(1)$. In addition (as will be presented subsequently) the covariance $P(1)$ will include a linearization to second order in $u(0)$. In [1,2], $P(1)$ was linearized to first order. It is believed that linearizations to second order are necessary to better account for the nonlinearity in $P(1)$ and $\hat{\theta}(1)$ of (3.3) and in $u(0)$ of (3.7) and (3.8). In addition a nonlinear Newton algorithm is used in the second order approximation.

Linearization of (3.3) about the nominal $\bar{\theta}(1) = A\hat{\theta}(0)$ and $\bar{P}(1)$ using the nominal $\bar{u}(0)$ results in

$$\begin{aligned} J^*(1) = {} & J^*[1,\,\hat{\theta}(0),\,\bar{P}(1)] + \frac{\partial J^*(1)}{\partial\hat{\theta}(1)}\,[\hat{\theta}(1) - A\hat{\theta}(0)] \\ & + \frac{1}{2}[\hat{\theta}(1) - A\hat{\theta}(0)]'\,\frac{\partial^2 J^*(1)}{\partial\hat{\theta}^2(1)}\,[\hat{\theta}(1) - A\hat{\theta}(0)] \\ & + \sum_{\ell=1}^{n}\sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\partial J^*(1)}{\partial P_{i,j}^\ell(1)}\,[P_{i,j}^\ell(1) - \bar{P}_{i,j}^\ell(1)] \end{aligned} \tag{3.13}$$

where the superscript ℓ represents the covariance matrix associated with the ℓ^{th} row of parameters and $P_{i,j}(1)$ is the i-j th element of the covariance matrix $P(1)$, m being the number of unknown parameters.

Using (3.6) the expected value of (3.13) is

$$\begin{aligned} E[J^*(1)\,|Y^o] = {} & J^*[1,\,\hat{\theta}(0),\,\bar{P}(1)] \\ & + \frac{1}{2}\,\mathrm{tr}\Big[\frac{\partial^2 J^*(1)}{\partial\hat{\theta}^2(1)}\,K(1)\;E\{\nu(1)\nu'(1)\,|Y^o\}K'(1)\Big] \\ & + \sum_{\ell=1}^{n}\sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\partial J^*(1)}{\partial P_{i,j}^\ell(1)}\,[P_{i,j}^\ell(1) - \bar{P}_{i,j}^\ell(1)] \end{aligned} \tag{3.14}$$

Using (3.7), (3.8) and the innovation covariance

$$E\{\nu_\ell(1)\;\nu_\ell'(1)\,|Y^o\} = H(1)P^\ell(0)H'(1) + W_\ell \tag{3.15}$$

(3.14) can be written as

$$\begin{aligned} E[J^*(1)\,|Y^o] = {} & J^*[1,\,\hat{\theta}(0),\,\bar{P}(1)] \\ & + \sum_{\ell=1}^{n}\sum_{i=1}^{m}\sum_{j=1}^{m}\Bigg\{-\frac{1}{2}\,\frac{\partial}{\partial\hat{\theta}_i(1)}\,\frac{\partial J^*(1)}{\partial\hat{\theta}_j(1)}\Big[P_{i,j}^\ell(1) - AP_{i,j}(0)A' \\ & - V_{i,j}^\ell\Big] + \frac{\partial J^*(1)}{\partial P_{i,j}^\ell(1)}[P_{i,j}^\ell(1) - \bar{P}_{i,j}^\ell(1)]\Bigg\} \end{aligned} \tag{3.16}$$

The expected future cost (3.16) is shown to be a function of the predicted covariance $P_{i,j}^\ell(1)$ with a multiplier given by the sensitivity $\frac{\partial J^*(1)}{\partial P_{i,j}^\ell(1)}$ and $\frac{\partial}{\partial\hat{\theta}_i(1)}\frac{\partial J^*(1)}{\partial\hat{\theta}_j(1)}$. Since the covariance $P_{i,j}^\ell(1)$ depends on the control $u(0)$ the control has the dual effect. It should be noted that the importance of the dual effect depends upon the sensitivity of the expected future cost with respect to both the covariance and parameter estimate.

The optimal control $u(0)$ can be computed by minimization of (3.2) using (3.16). Since $P_{i,j}^\ell(1)$ is nonlinear in $u(0)$ a numerical search procedure is required. This is accomplished using a second order linearization in u(0).

Thus (3.8) is linearized to second order about the control $u^I(0)$, which is in the vicinity of the optimal control.

$$P^{\ell}_{i,j}(1) \simeq \bar{P}^{\ell}_{i,j}(1) + \frac{\partial P^{\ell}_{i,j}(1)}{\partial u(0)}\bigg|_{u^I(0)} [u(0)-u^I(0)]$$

$$+ \frac{1}{2}[u(0)-u^I(0)]' \frac{\partial^2 P^{\ell}_{i,j}(1)}{\partial u^2(0)}\bigg|_{u^I(0)} [u(0)-u^I(0)] \qquad (3.17)$$

The expected future cost as given by (3.16) and (3.17) is quadratic in u(0) and thus a closed form solution $u^*(0)$ is obtained by minimization of (3.2).

The optimal dual control $u^*(0)$ can now be computed from (3.2) using (3.16) and (3.17). It is obtained by solving

$$\frac{\partial}{\partial u(0)} E\{x'(1)Q\,x(1) + u'(0)Ru(0) + J^*(1)|Y^o\} = 0 \qquad (3.18)$$

The optimal $u^*(0)$ is thus

$$u^*(0) = -[\hat{B}'(0)Q\hat{B}(0) + \sum_{\ell=1}^{n} (q_\ell P^{\ell}_{B}(0) + F_\ell) + R]^{-1}$$

$$[\hat{B}'(0)Q\hat{c}(0) + \sum_{\ell=1}^{n} (q_\ell P^{\ell}_{Bc}(0) + f_\ell)] \qquad (3.19)$$

where the matrix F_ℓ and the vector f_ℓ are

$$F_\ell = \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{1}{2}\left(\frac{\partial J^*(1)}{\partial P^{\ell}_{i,j}(1)} - \frac{1}{2}\frac{\partial}{\partial \hat{\theta}^{\ell}_i(1)} \frac{\partial J^*(1)}{\partial \hat{\theta}^{\ell}_j(1)}\right)$$

$$\frac{\partial}{\partial u(0)} \frac{\partial P^{\ell}_{i,j}(1)}{\partial u(0)}\bigg|_{u^I(0),\hat{\theta}(0),\bar{P}(1)} \qquad (3.20)$$

$$f_\ell = \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{1}{2}\left(\frac{\partial J^*(1)}{\partial P^{\ell}_{i,j}(1)} - \frac{1}{2}\frac{\partial}{\partial \hat{\theta}^{\ell}_i(1)} \frac{\partial J^*(1)}{\partial \hat{\theta}^{\ell}_j(1)}\right)\left(\frac{\partial P^{\ell}_{i,j}(1)}{\partial u(0)} - \right.$$

$$\left. - \frac{\partial}{\partial u(0)} \frac{\partial P^{\ell}_{i,j}(1)}{\partial u(0)}\; u^I(0)\right)\bigg|_{u^I(0),\hat{\theta}(0),\bar{P}(1)} \qquad (3.21)$$

Initially the nominal value of $\bar{u}(0)$ is computed from (3.19) with F_ℓ and f_ℓ equal to zero. Then a gradient search is performed until in the vicinity of the optimal $u^*(0)$. Then (3.19) - (3.21) are used until convergence is achieved. This iteration procedure is essentially Newton's method for minimization of a nonlinear function. The gradient search is used because the stochastic cost in (3.2) being minimized is a high order nonlinear equation and the gradient procedure is used until $u^I(0)$ is in the vicinity of the minimum before switching to the Newton method. The nominal covariance $\bar{P}^{\ell}(1)$ is computed from((3.7) - (3.11)) with $u(0) = \bar{u}(0)$. The sensitivity (partials) in (3.20) and (3.21) of the cost $J^*(1)$ are computed from partial derivatives of $J^*(1)$ (3.3) and $P^{\ell}(1)$ (3.7) - (3.9) evaluated at the nominal. The partials of the covariance are evaluated at $u^I(0)$ which is evaluated at the previous iteration I.

The approximate two-step ahead dual control of (3.19) - (3.21) can be interpreted as a modification to the cautious controller by the terms F_ℓ and f_ℓ. These terms depend upon the sensitivity of the future nominal cost $J^*(1)$ with respect to the parameters $\hat{\theta}^{\ell}_i(1)\hat{\theta}^{\ell}_j(1)$ for all i,j and their covariance $P^{\ell}_{i,j}(1)$ for each row ℓ of parameters. Whenever these sensitivities are large the terms F_ℓ and f_ℓ will be significant (that is the dual effect will be important). Thus the sensitivities take into account in the control solution the sensitivity of the nominal future cost due to parameter variation and uncertainty. The larger this sensitivity the more important will be the dual affect.

The resulting dual controller (3.19) exhibits a robustness property with respect to parameter variations and uncertainty of the future cost by including a term which appears in the denominator of the dual controller. In addition, a probing term appears in the numerator.

4. SCALAR EXAMPLE WITH ONE UNKNOWN PARAMETER

To further understand the dual control solution a scalar example with one unknown parameter b is presented. The approximate dual control solution for this scalar case using Q = 1, R = 0, is given by (3.19) - (3.21) with $P_{i,j}(1)$ and $\hat{\theta}(0)$ being replaced by $P_b(1)$ and $\hat{b}(0)$ respectively.

The partials required in the control law are

$$\frac{\partial J^*(1)}{\partial P_b(1)}\bigg|_{\hat{b}(0),\bar{P}_b(1)} = \frac{c^2\hat{b}^2(0)}{(\hat{b}^2(0)+\bar{P}_b(1))^2} \qquad (4.1)$$

$$\frac{\partial^2 J^*(1)}{\partial \hat{b}(1)\,\partial \hat{b}(1)}\bigg|_{\hat{b}(0),\bar{P}_b(1)} = -2c^2\bar{P}_b(1)\left|\frac{\bar{P}_b(1)-3\hat{b}^2(0)}{(b^2(0)+\bar{P}_b(1))^3}\right| \qquad (4.2)$$

$$\frac{\partial P_b(1)}{\partial u(0)}\bigg|_{u^I(0)} = -\frac{2P_b^2(0)W\,u^I(0)a^2}{(P_b(0)u^{I^2}(0)+W)^2} \qquad (4.3)$$

$$\frac{\partial^2 P_b(1)}{\partial u(0)\,\partial u(0)}\bigg|_{u^I(0)} = -2P_b^2(0)W\left|\frac{W-3P_b(0)u^{I^2}(0)}{(P_b(0)u^{I^2}(0)+W)^3}\right|a^2 \qquad (4.4)$$

where the nominal $\bar{u}(0)$ and $\bar{P}_b(1)$ are

$$\bar{u}(0) = -\frac{\hat{b}(0)c}{\hat{b}^2(0)+P_b(0)} \qquad (4.5)$$

$$\bar{P}_b(1) = \frac{a^2P_b(0)W}{P_b(0)\bar{u}^2(0)+W} + V \qquad (4.6)$$

The parameter estimate $\hat{b}(0)$ and $P_b(0)$ are computed using data up to k = 0 (i.e. y(0)).

The expected future cost based upon the linearization of (3.16) is

$$E\{J^*(1)|Y^o\} = c^2 - c^2\frac{\hat{b}^2(0)}{\hat{b}^2(0)+\bar{P}_b(1)} - \frac{1}{2}\frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)}(P_b(1)$$

$$- a^2P_b(0)-V) + \frac{\partial J^*(1)}{\partial P_b(1)}(P_b(1)-\bar{P}_b(1)) \qquad (4.7)$$

4.1 Evaluation of the Cautious Controller

The performance of the cautious controller can be evaluated using (3.2) with u(0) evaluated at the nominal

$$J(0) = [E\{x^2(1)|Y^o\} + E\{J^*(1)|Y^o\}]_{u(0)=\bar{u}(0)} \qquad (4.8)$$

The first term in (4.8) represents the expected cost at k = 1 and the second term in (4.8) represents the expected future cost at k = 2 using the cautious control at k = 2 (i.e. u(1)) and using the cautious control at k = 1 (i.e. $u(0)=\bar{u}(0)$). (4.8) is evaluated using data Y^o.

Using (4.1) - (4.7), (4.8) becomes,

$$J(0) = c^2 - \frac{\hat{b}^2(0)c^2}{\hat{b}^2(0)+P_b(0)} + c^2 - c^2\frac{\hat{b}^2(0)}{\hat{b}^2(0)+P_b(1)}$$

$$+ \frac{1}{2}\frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)} \cdot \frac{a^2P_b^2(0)\bar{u}^2(0)}{P_b(0)\bar{u}^2(0)+W} \qquad (4.9)$$

The last term in (4.7) is zero since $P_b(1)$ evaluated at the nominal control (i.e. cautious control) equals $\bar{P}_b(1)$. The first two terms in (4.9) represent the average cost at step k = 1 and the last three terms represent the expected future cost at k = 2 using the cautious control.

A simple example can be used with (4.9) to demonstrate when the cautious control is expected to behave poorly.

Assume a scalar example with one unknown b parameter and let

$$\hat{b}(0) = .05 \ , \ P(0) = .5 \ , \ a = 1.0 \qquad V = .1 \ , \ W = .1 \ , \ c = 1 \tag{4.10}$$

The expected cost at k = 1 and k = 2 is computed from the nominal, $\bar{u}(0)$, $\bar{P}_b(1)$ and $\frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)}$ which yields

$$\bar{u}(0) = -.1 \ , \ \bar{P}_b(1) = .575 \ , \ \frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)} = -3.47 \tag{4.11}$$

and

$$J(0) \approx c^2 + c^2 \ , \ c = 1 \tag{4.12}$$

Thus the cautious control applied at k = 0 results in no reduction in the cost at k = 1 due to large uncertainty P(1) and also no reduction in the future expected cost since $\bar{u}(0)$ is small and no improvement in parameter accuracy occurs at step k = 1.

4.2 Evaluation of the Dual Controller

The dual controller of (3.19) - (3.21), (4.1) - (4.6) can be evaluated by computing the average cost of (4.8) using the covariance

$$P_b(1) = \frac{a^2 P_b(0) W}{P_b(0) u^2(0) + W} + V \tag{4.13}$$

The expected future cost (4.7) reduces to

$$E\{J^*(1)|Y^o\}\Big|_{u^*(0)} = c^2 - c^2 \frac{\hat{b}^2(0)}{\hat{b}^2(0) + \bar{P}_b(1)} + \frac{1}{2} \frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)} \frac{a^2 P_b^2(0) u^{*2}(0)}{P_b(0) u^{*2}(0) + W} - \frac{\partial J^*(1)}{\partial P_b(1)} \left(\frac{a^2 P_b^2(0) u^{*2}(0)}{P_b(0) u^{*2}(0) + W} - \frac{a^2 P_b^2(0) \bar{u}^2(0)}{P_b(0) \bar{u}^2(0) + W} \right) \tag{4.14}$$

and the total expected cost at k = 1 and k = 2 using (4.8) is

$$J^*(0) = E\{x^2(1)|Y^o\}\Big|_{u^*(0)} + E\{J^*(1)|Y^o\}\Big|_{u^*(0)} \tag{4.15}$$

where

$$E\{x^2(1)|Y^o\}\Big|_{u^*(0)} = c^2 + 2\hat{b}(0) u^*(0) c + (\hat{b}^2(0) + P_b(0)) u^{*2}(0) \tag{4.16}$$

Examination of (4.14) shows that the dual control can reduce the expected future cost over the cautious control since the last two expressions in (4.14) can be negative if $u^{*2}(0) > \bar{u}^2(0)$. Thus the dual property can have a desirable effect on the future cost.

The cost $J^*(0)$ is computed using the scalar example previously discussed for the cautious controller. A search procedure is used on (4.15) using (4.14) and (4.16) with the parameter values from (4.10), and $u^*(0)$ is iterated until in the vicinity of the minimum yielding

$$\frac{\partial J^*(1)}{\partial P_b(1)}\Big|_{\bar{u}(0)=-.1} = .0075 \ , \ \frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)}\Big|_{\bar{u}(0)=-.1} = -3.47 \ ,$$

$$\frac{\partial P_b(1)}{\partial u(0)}\Big|_{u^I(0)=-.6} = .382 \ , \ \frac{\partial^2 P_b(1)}{\partial u^2(0)}\Big|_{u^I(0)=-.6} = +1.0 \ ,$$

$$F_\ell = .87 \ , \ f_\ell = .85 \tag{4.17}$$

The above sensitivities (4.17) were evaluated in the vicinity of the optimal $u^I(0) = -.6$ and $P_b(1) = .278$. The dual control $u^*(0)$ using $u^I(0) = -.6$, c=1 is

$$u^*(0) = - \frac{\hat{b}(0) c + .85}{\hat{b}^2(0) + P_b(0) + .87} = -.62 \tag{4.18}$$

The corresponding future expected cost using (4.14) and (4.17) is

$$E\{J^*(1)|Y^o\}\Big|_{u^*(0)} \approx c^2 + \frac{1}{2} \frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)} \frac{P_b^2(0) u^{*2}(0)}{P_b(0) u^{*2}(0) + W} \approx .442 \ c^2 \ , \ c = 1 \tag{4.19}$$

The result of this example shows that the dual control of (4.18) reduces the expected future cost to 44% of the original c^2 with no control. The cautious control resulted in no reduction of the future cost. The terms responsible for the improvement with dual control are the second order sensitivities $\frac{\partial^2 P(1)}{\partial u^2(0)}$ and $\frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)}$.

The dual control of (4.18) differs from the cautious control (4.11) by the terms $F_\ell = .87$ in the denominator and $f_\ell = .85$ in the numerator. The denominator term in effect provides more "caution" whereas the numerator term is an additive probing effect. The term F_ℓ provides a "robustness" property in that the sensitivity of the future cost to parameter uncertainties as they appear in the controller (i.e. $\hat{b}^2(0)$) are minimized. Thus a new interpretation of the dual control is that it contains robustness and learning (via probing). These concepts are applicable to the multivariable dual controller in (3.19) - (3.21).

5. SIMULATION RESULTS

Performance was evaluated from 100 Monte Carlo runs for the following controllers where b(0) was set to b(0) with covariance $P_b(0)$: 1) Cautious Controller 2) FOD 3) SOD

The above algorithms were tested for two cases:

a) Time varying case, $b(0) = .05$, $P_b(0) = 1.0$, $V = .1$, $c = 1.0$, $W = .01$ and $W = .1$, $a = 0.9$

b) Constant case, with $b(0) = .05$, $P_b(0) = 1.0$, $V = 0$, $c = 1.0$, $W = .01$ and $W = .1$, $a = 1.0$

Example a

Table 1 summarizes the results of the simulation runs. All three algorithms were tested on this example for two different levels of measurement noise covariance, W = .01 and W = .1. 100 Monte Carlo runs were performed each of 40 time steps. For each run, an average cost was computed over 40 time steps and then the averages over 100 runs are tabulated in Table 1 and Table 2. The tables clearly indicate that the SOD yields the least cost. The dual effect shows a larger improvement for larger measurement noise (i.e. W = .1). Run numbers 7 and 14 of the 100 Monte Carlo runs were selected for plotting. The cost and parameter value are plotted in Figures 1 through 4. It is evident that the second order dual improves upon the other two on the average.

Example b

In this case the true parameter was close to zero (i.e., b(0) = .05) but constant. Table 2 summarizes the result. The average cost obtained by the SOD is much lower than the other two. The SOD always exhibited excellent convergence whereas the other controllers performed poorly. In addition the new controller consistently avoided turn off and burst [5]. This was an important common feature in all the Monte Carlo runs. Runs 26 and 80 are plotted in Figures 5 and 6 respectively, as typical examples.

The simulation study has shown that the new dual controller improves upon the cost on the average. The magnitude of the improvement on the average appears to be relatively small for the noise levels used. However, the real advantage of the new dual controller is the improvement in those instances where the cautious controller and the FOD [1,2] yields unacceptable results. Although the FOD [1,2] shows improvement over the cautious controller, it has been found to be unacceptable at many time points.

6. CONCLUSION

A new adaptive dual control solution based upon the sensitivity functions of the expected future cost has been presented. This controller (SOD) takes into account the dual effect better by performing the second order Taylor series expansion of the expected future cost. The form of this controller is a modification of the one step cautious controller. The FOD of [1,2] did not have the denominator correction term like the present one. This adds stability to the new control design. Simulation results of a scalar model have shown the improvement obtained using the new dual algorithm.

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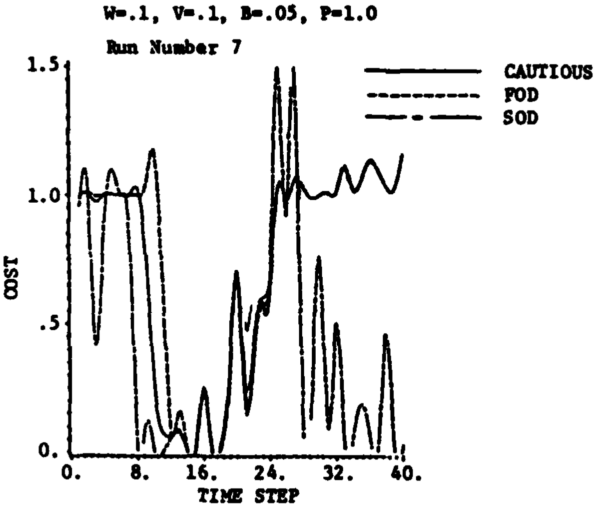

Fig. 1. Time history of cost comparing the SOD, FOD, and the cautious controller (Time varying parameter case: Run No. 7 from 100 Monte Carlo Runs).

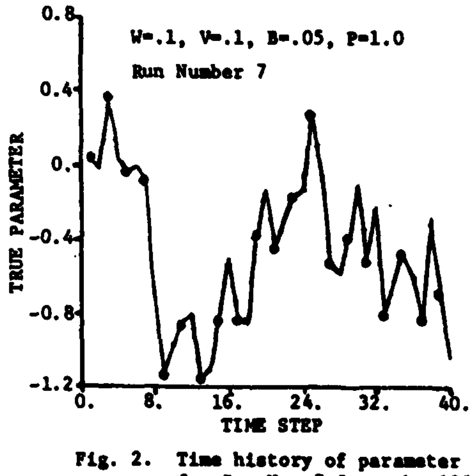

Fig. 2. Time history of parameter for Run No. 7 from the 100 Monte Carlo Runs (Time Varying Case).

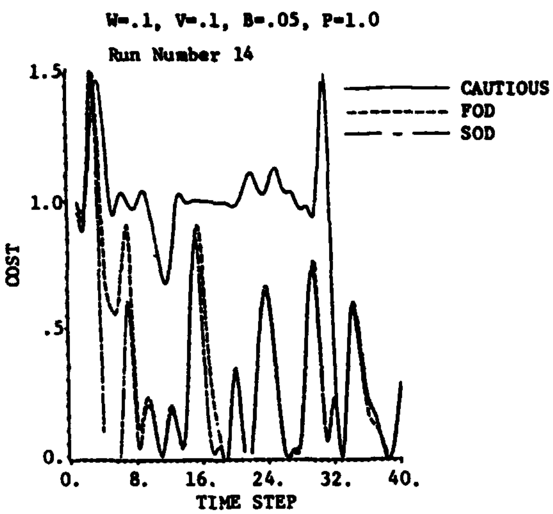

Fig. 3. Time history of cost comparing the SOD, FOD, and the cautious controller (Time varying parameter case: Run No. 14 from 100 Monte Carlo Runs)

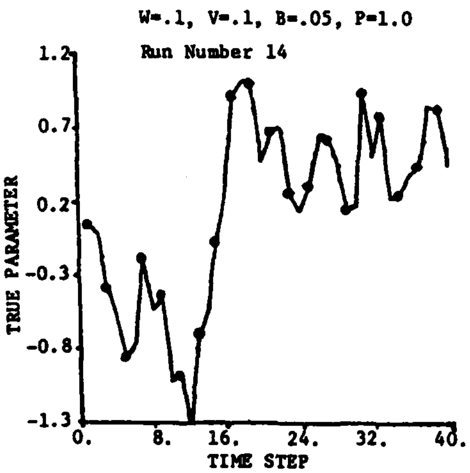

Fig. 4. Time history of parameter for Run No. 14 from 100 Monte Carlo Runs (Time Varying Case).

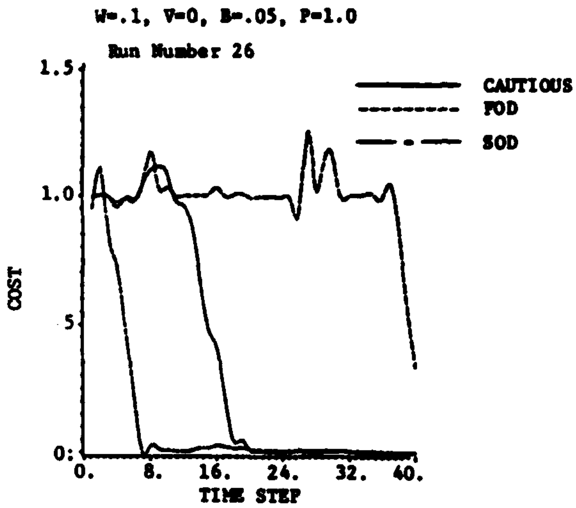

Fig. 5. Time history of cost comparing the SOD, FOD, and the cautious controller (Constant parameter case: Run No. 26 from 100 Monte Carlo Runs).

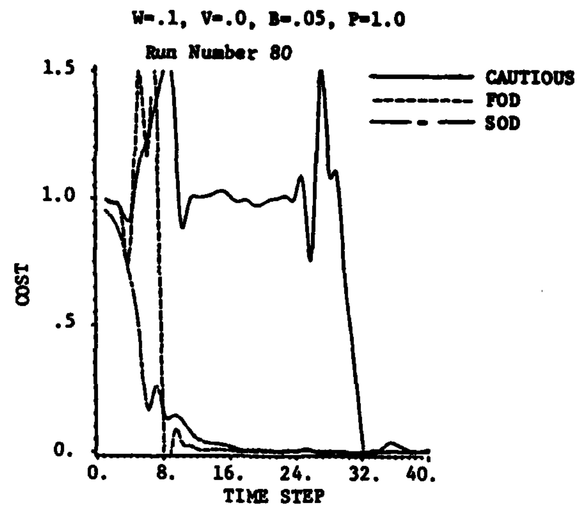

Fig. 6. Time history of cost comparing the SOD, FOD, and the cautious controller (Constant parameter case: Run No. 80 from 100 Monte Carlo Runs).

Measurement Noise Covariance W	Average Cost		
	Cautious	First Order Dual	Second Order Dual
.01	.475	.469	.458
.1	.623	.608	.514

Table 1. Average Cost for the three controllers on the time varying parameter model ($b(0)=.05$, $P_b(0)=1$, $V=.1$, $c=1$)

Measurement Noise Covariance W	Average Cost		
	Cautious	First Order Dual	Second Order Dual
.01	.109	.087	.069
.1	.359	.250	.142

Table 2. Average Cost for three controllers on the Constant Parameter Model ($b(0)=.05$, $P_b(0)=1$, $V=0$, $c=1$)

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